GEO4-1415 Data processing and inverse theory

Tentamen - 7 Nov 2019 - 13h30-16h00 - RUPPERT-B

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The numbers in () indicate percentage marks for the evaluation. No documents are allowed during the examination. Please write clearly and feel free to give your answers in Dutch, English, French, German or Luxembourgish.

1. (20) Consider the following filter defined in the Z-domain:

$$F(z) = \frac{(1-z)(1+z)}{1+az^2} \tag{1}$$

where $a = 0.5$.

Imagine you have an input wavelet $x(t)$ and you filter it with the impulse response $f(t)$ to get an output $y(t)$. Noting that the filter defined in equation (1) is recursive, write the recursive relation for $y(t)$ in the time domain.

Is this a high-, low-, band-pass or band-rejection filter? To answer this you should sketch the amplitude response of the filter $F(z)$. Hint: use the following table

θ	0	$-\pi/4$	$-\pi/2$	$-3\pi/4$	$-\pi$
ν					
$\cos 2\theta$					
$\sin 2\theta$					
$F(z)$					

2. (30) We give the wavelet $a_t = (4, 2, 1)$. Calculate the corresponding inverse wavelet of length 2, denoted $b_t = (x, y)$, by polynomial division in the Z-domain.

Give a general expression for the energy of the error wavelet and calculate the energy of the error.

Now solve the same deconvolution problem using a Wiener filter. Calculate again the energy of the error. Which one has the smallest error and why?

3. (50) Consider the same deconvolution problem as in question (2), but now write the operation $a_t * b_t = \delta(t)$ as a linear system $d = Gm$.

Solve this system using a damped least squares solution given by

$$\tilde{m} = (G^t G + \theta I)^{-1} G^t d = G^t (GG^T + \theta I)^{-1} d \tag{2}$$

Choose wisely one of the 2 formulations and justify your choice. Solve the system for $\theta = 0$, $\theta = 1$ and $\theta = \infty$

Now calculate the resolution matrix for each value of θ.

Discuss the properties of the solution using the resolution matrix and the expression of energy of the error given in question (2).

Hint: Give general expressions as a function of θ first before giving numerical answers.

Imagine solving the system above using SVD. For which value of θ would you find the same answer as SVD. Justify your response. You don't necessarily have to do the SVD.

Good luck.