

# Midterm Exam

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October 4, 2012

## 1 Theory

In continuum mechanics, stress is a measure of the internal forces acting within a deformable body. It is represented mathematically by a tensor that can be represented in 2D and 3D by  $2 \times 2$  or  $3 \times 3$  matrices respectively:

$$\boldsymbol{\sigma}_{2D} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \quad \boldsymbol{\sigma}_{3D} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

One can define the trace of the stress tensor which is given by

$$Tr(\boldsymbol{\sigma}_{2D}) = \sigma_{11} + \sigma_{22}$$

$$Tr(\boldsymbol{\sigma}_{3D}) = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

Also, the determinant of the tensor is given by

$$\begin{aligned} D(\boldsymbol{\sigma}_{2D}) &= \sigma_{11}\sigma_{22} - \sigma_{21}\sigma_{12} \\ D(\boldsymbol{\sigma}_{3D}) &= \sigma_{11} * (\sigma_{22}\sigma_{33} - \sigma_{32}\sigma_{23}) \\ &\quad - \sigma_{21} * (\sigma_{12}\sigma_{33} - \sigma_{32}\sigma_{13}) \\ &\quad + \sigma_{31} * (\sigma_{12}\sigma_{23} - \sigma_{22}\sigma_{13}) \end{aligned}$$

Applying a stress  $\boldsymbol{\sigma}$  on a solid, the force exerted on its surface defined by a normal vector  $\mathbf{n}_{2D} = (n_1, n_2)$  or  $\mathbf{n}_{3D} = (n_1, n_2, n_3)$  is given by

$$\mathbf{f} = \boldsymbol{\sigma} \cdot \mathbf{n}$$

or,

$$\mathbf{f}_{2D} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \sigma_{11}n_1 + \sigma_{12}n_2 \\ \sigma_{21}n_1 + \sigma_{22}n_2 \end{pmatrix} \quad \mathbf{f}_{3D} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} \sigma_{11}n_1 + \sigma_{12}n_2 + \sigma_{13}n_3 \\ \sigma_{21}n_1 + \sigma_{22}n_2 + \sigma_{23}n_3 \\ \sigma_{31}n_1 + \sigma_{32}n_2 + \sigma_{33}n_3 \end{pmatrix}$$

The stress tensor is said to be symmetric when for all couples  $(i, j)$

$$\sigma_{ij} = \sigma_{ji}$$

One can obtain the symmetric part of a tensor by computing:  $\boldsymbol{\sigma}^s = \frac{1}{2}(\boldsymbol{\sigma} + \boldsymbol{\sigma}^T)$  where  $\boldsymbol{\sigma}^T$  is the transpose of the stress tensor given by  $\sigma_{ij}^T = \sigma_{ji}$ .

## 2 Assignment

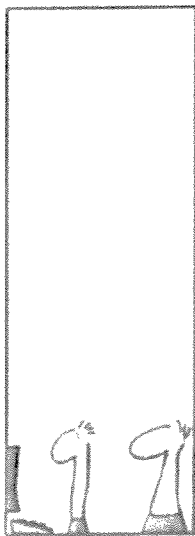
Write a fortran program according to the following specifications:

1. (1pt) declare the following arrays
  - `force` : 1D allocatable array
  - `stress` : 2D allocatable array
  - `stressT` : 2D allocatable array
  - `stress_sym` : 2D allocatable array
2. (1pt) read from the standard input the number of dimensions `ndim`
3. (1pt) allocate the arrays to their correct sizes

4. (1pt) fill the stress tensor with random numbers
5. (2pt) write a simple subroutine which takes the two arrays `stress` and `stressT` as arguments and returns the transpose of `stress` in `stressT`
6. (1pt) compute the symmetric part of the stress tensor and store it in `stress_sym`
7. (1pt) compute the trace `Tr` and the determinant `Det` of the stress tensor
8. (1pt)  $\mathbf{n}_{2D} = (-1, -1)$  in 2D or  $\mathbf{n}_{3D} = (-\cos(\theta), \sin(\theta), \tan(\theta))$  with  $\theta = 0.5$  in 3D. Compute the force exerted on this surface.
9. (1pt) write in file `measurement.dat` the number of dimensions, the trace, the determinant, and the computed force.

Recommendations:

- Keep in mind that the code has to work for `ndim=2` or `3`, and that it should not work for `nd=-1`, or `nd=4`. (`If-then-else` constructions might be of use here throughout the program)
- Every single used variable has to be defined.
- Comment your code appropriately.
- Points will be deducted for unclear/unreadable statements.
- Every single used variable has to be defined (I insist).



Good luck